Using DNS Results of Stratified and Non-stratified Turbulence to Measure Effective Eddy-Viscosity

S. Khani

1 Introduction

Direct numerical simulation (DNS) has an intrinsic limitation that confines this approach to be used in moderate (small) Reynolds numbers. The atmospheric and oceanic turbulence, in which Reynolds numbers are of the order 10^8 or higher, need, therefore, an alternative approach for numerical simulations. Large-eddy simulation (LES) is a numerical technique that takes away resolving all scales but the large-scale motions through a subgrid-scale (SGS) model. Hence, the accurate predictions of velocity, temperature, and pressure depend on SGS (or eddy-viscosity) models.

In this (very short) paper, I measure effective eddy-viscosity using DNS results of decaying stratified and non-stratified turbulence. Moreover, measured eddy-viscosities are compared to the theoretical one, which is suggested by [1].

2 The Governing Equations

Applying the boussinesq approximation to the Navier-Stokes equations results in the governing equations of motion for stratified flow [2]. Let us assume the background density gradient $d\rho_0/dz$ is constant, therefore, we can write the momentum equation in the spectral space as follows:

$$(\frac{\partial}{\partial t} + \nu k^2)\hat{u}_j(\boldsymbol{k},t) + \frac{g}{\rho_{00}}\hat{\rho}'(\boldsymbol{k},t)\boldsymbol{e}_z = -ik_m P_{jr} \sum_{\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{k}} \hat{u}_r(\boldsymbol{p},t)\hat{u}_m(\boldsymbol{q},t) \equiv F_j(\boldsymbol{k},t). \quad (1)$$

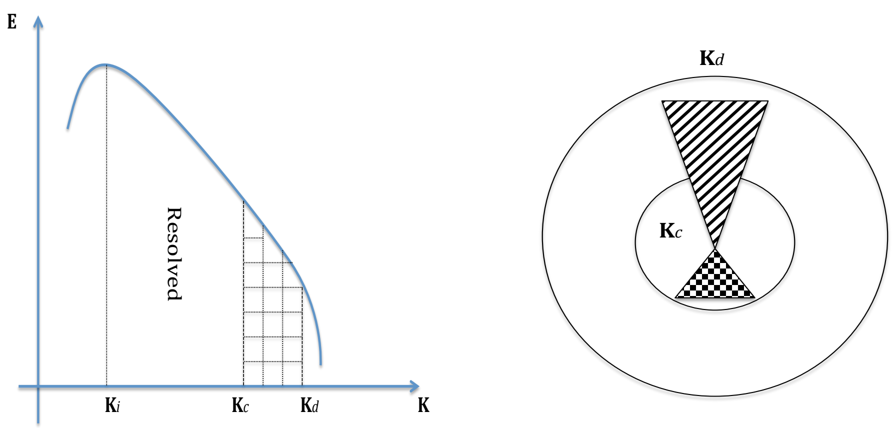

Figure 1: Schematic sketches of energy spectrum and triad interactions

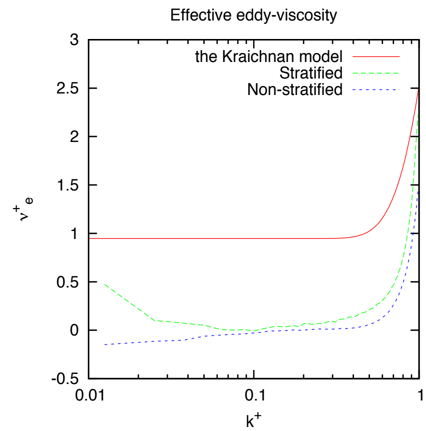

Figure 2: Effective eddy-viscosities, where $\nu_e^+ = \nu_e/\nu$ and $k^+ = k/k_c$, with $k_c = 40\text{m}^{-1}$. Solid-line: the Kraichnan model [3], long-dashed: stratified, short-dashed: non-stratified eddy-viscosity.

If we define a test cutoff k_c inside the resolved wavenumbers, we can decompose F_j into $\overline{F}_j$ – in which the nonlinear interactions of the momentum equation have been restricted between those wavenumbers such that $|\boldsymbol{q}|, |\boldsymbol{p}| \leq k_c$ – and F_j^s – and so either $|\boldsymbol{p}|$ or $|\boldsymbol{q}|$ is above k_c. Therefore, we can measure the effective eddy-viscosity as follows:

$$\nu_e(k|k_c,t) = -\frac{F_j^s(k|k_c,t)}{k^2\overline{u}_j(k|k_c,t)} = -\frac{F_j(k|k_c,t) - \overline{F}_j(k|k_c,t)}{k^2\overline{u}_j(k|k_c,t)}, \tag{2}$$

where equation (2) has been averaged over spheres of radius k.

3 Methodology

A cube of side-length $\mathcal{L} = 4\pi\ell$ with periodic boundary conditions has been considered. The Taylor-Green (TG) vortices in addition to low-level noise have been designed to as initial condition. The spectral transform method has been used to compute the spacial derivatives.

4 Results

Figure 2 shows measured eddy-viscosity for stratified and non-stratified turbulence. It is obvious that the theoretical model has a constant plateau away from the cutoff and a cusp-like behaviour near to k_c. However, the measured eddy-viscosity has almost zero plateau, for the stratified case. On the other hand, stratified eddy-viscosity presents a higher plateau respect to non-stratified one but still smaller than Kraichnan's.

References

[1] Kraichnan R. H., J. of the Atmospheric Sciences **33** (1521-1536), 1976.

[2] Kundu P. K. , and Cohen I. M., Academic press, Burlington, USA, 2008.

[3] Lesieur M. , and Rogallo R., Phys. Fluids A **1(4)** (718-722), 1989.