The Stochastically Forced Regularized Long-Wave Equation

Marek Stastna and Kris Rowe

September 26, 2012

Many studies have been conducted on stochastically forced, nonlinear, dispersive, wave equations. Two major issues that are common to these studies are that they use a white noise forcing with limited spatial extent (i.e. a delta function), often in combination with a wave equation whose dispersion relation is badly behaved for small scales (large wave numbers).

We have reexamined this problem with the simplest nonlinear wave equation whose dispersion relation is well behaved for all scales, as well as a coloured noise forcing with finite spatial extent. This model is given by the following forced *regularized long-wave equation*,

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \beta\frac{\partial^3 u}{\partial x^2 t} - u\frac{\partial u}{\partial x} = \tilde{\alpha}\,\mathrm{sech}\left(\frac{x}{0.5}\right)^2, \tag{1}$$

where c is the linear advection velocity, β is the dispersion parameter, and $\tilde{\alpha}$ is a random variable determined from coloured noise. The parameter values used were $c = 1$ and $\beta = 0.1$. The dispersion relation for this model is

$$\omega = \frac{1}{1 + \beta k^2}. \tag{2}$$

The model equation was solved using a Fourier based spectral method and Bartosch's red noise algorithm. A number of numerical experiments were conducted, each time increasing the memory of the coloured noise. Three sub-experiments were conducted within each experiment, removing the effects of nonlinearity and then dispersion for comparison purposes. In each experiment a time-series was extracted downstream from the forcing region. Both the amplitude and the local slope of the waves were recorded.

Figure 1 displays statistics calculated from the amplitude time-series. The horizontal axis gives the length of the memory of the coloured noise, while the vertical axis gives the value of the statistic calculated. The circular markers display results for the full model, the square markers the results for the model in the absence of nonlinearity, and the star markers the results for the model in the absence of both nonlinearity and dispersion.

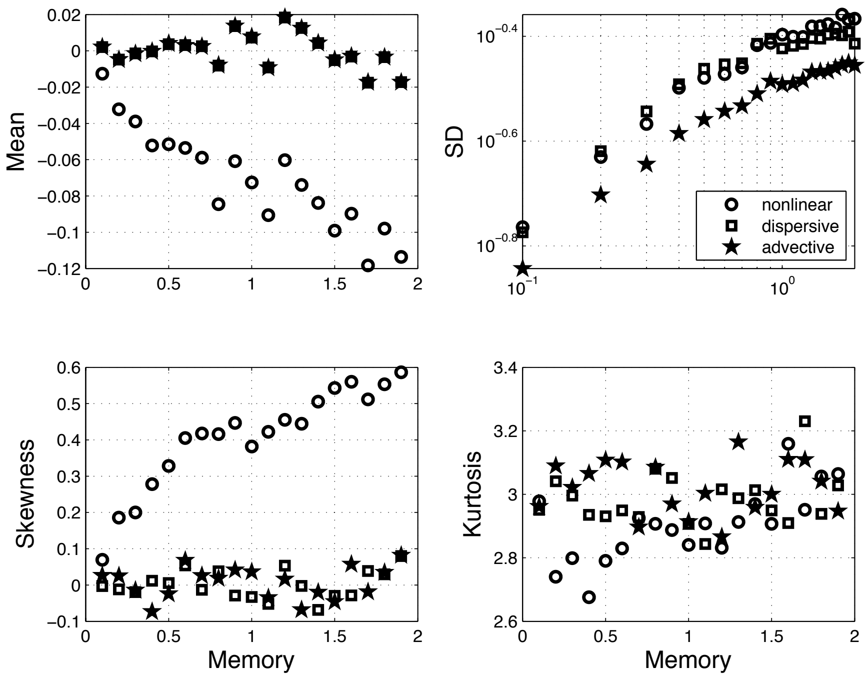

Figure 1: Statistics for the amplitude time-series