1 Fun with KH Billows

Following up on my precomp talk (shame on you for missing it). I'm going to talk a little bit about Kelvin-Helmholtz (KH) billows.

To briefly introduce the topic, KH billows form when a fluid undergoes some form of shear. Internal waves experiencing a strong shear flow undergo rapid breaking and overturn. The greater the shear the more rapid and interesting the formation of the billows. KH billows occur over a wide range of scales, from internal waves in ocean and lakes to atmospheric waves. This is a fairly well-studied problem (see the Caulfield and Peltier review for a recent synopsis of the work to date), and a classically well-posed problem.

Analytically the growth rate of KH billows can be derived by solving the Taylor-Goldstein equation:

$$(U-c)\left(\frac{d^2}{dz^2}-k^2\right)\hat{\phi}-U_{zz}\hat{\phi}+\frac{N^2}{U-c}\hat{\phi}=0$$

where $\hat{\phi}$ is the stream function, U is the background velocity, $c=c_r+ic_i$ is the (complex) eigenvalue, and everything else is as normal. The value of c_i gives the rate of growth of the unstable mode, so every growing mode is coupled with a decaying mode. This equation is used to determine a linear stability criterion for the Richardson number, $Ri=\frac{N^2}{U_z^2}>\frac{1}{4}$.

My (our) investigations revolve around confined KH billows - a lab-scale tube (1-2cm high) filled with light fluid over heavy fluid tilted at 45 degrees to induce shear flow. Lots of wonderful pictures are available in my pre-comp talk. We ran a few simulations of a 5 by 2 by 1cm box (x,y,z respectively) for initial investigations. Following up on that we decided to run a few 2D simulations (since the vast majority of the KH formation happens in two dimensions (Squire's theorem)). The followup investigations were run on plata, with 8 processors on a 768 by 128 (or 192) grid in x and z respectively.

4 simulations were performed in total, 2 with a domain of 10cm by 1cm, and 2 with a domain of 10cm by 2cm. Similarly there are 2 categories - centered pycnocline, and 25% from the top boundary pycnocline. Remember that the top and bottom boundaries are no-slip with a chebyshev vertical grid, which clusters points about the boundaries. What we see from the pycnocline 1cm vs off-centered 2cm tall simulations is that in the larger domain the billows have a chance to interract with each other. Also the smaller domain appears to induce more dissipation near the boundaries.

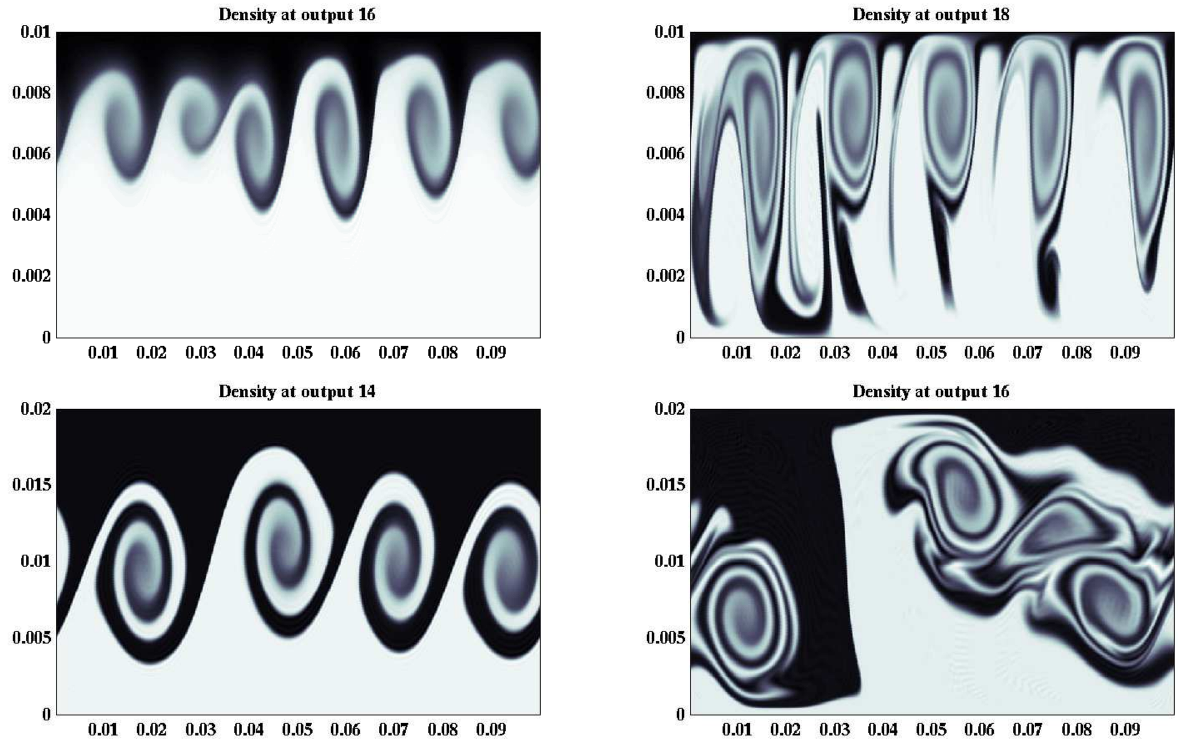

Figure 1: Some Densities of the 1cm (top) and 2 cm (bottom) domains. The top simulation is the off-center pycnocline, the bottom simulation is the centered pycnocline.

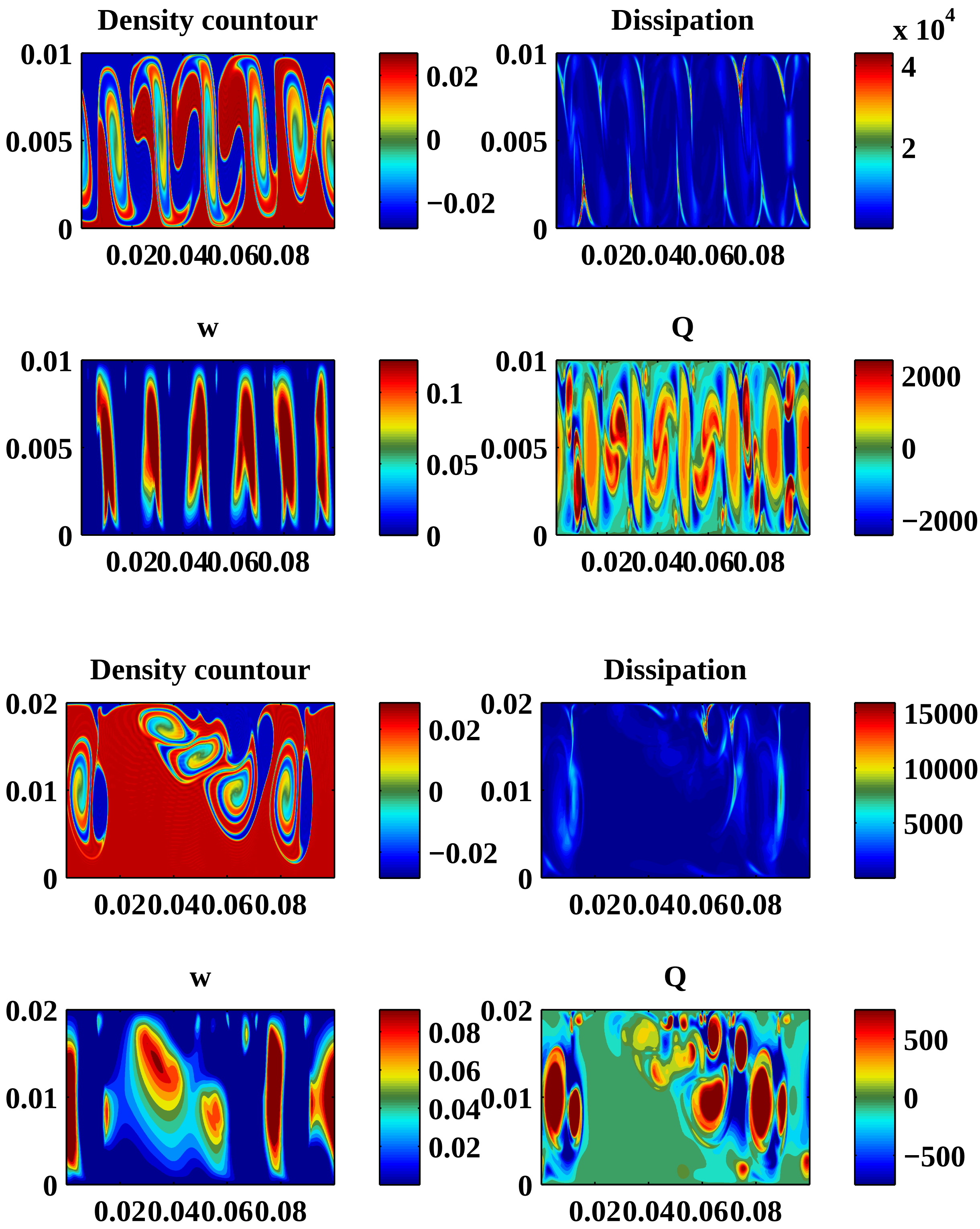

Figure 2: Some figures of some things. Density, Dissipation, and w are pretty self-explanatory, however it's worth noting that this Q variable may or may not make a lot of sense in 2D versus 3D, it might amount to just displaying vorticity... more thoughts to come.